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THE EXCITATION OF SYMMETRIC AND ANTISYMMETRIC LAMB WAVES IN A PIEZOELECTRIC STRIP BY SURFACE ELECTRODES†

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Solutions of the problems of the excitation by surface electrodes of symmetric and antisymmetric Lamb waves in a strip of piezoelectric material are obtained. The solutions of the problems are reduced to systems of singular integral equations in auxiliary functions, proportional to the jump in charge density at the electrodes. Approximate solutions of the systems of singular integral equations are obtained by the Bubnov–Galerkin method using Chebyshev polynomials of the first kind. The results of numerical calculations of the electro-elastic fields for a strip of cadmium sulphide are given.

THE PROBLEM of the excitation of oscillations in a rectangle was considered in [1], the dispersion relations of antisymmetric oscillations of a piezoelectric plate with uniform conditions on the surfaces were investigated in [2, 3], and the problem of the amplification of Lamb waves was considered in [4], where the velocities of symmetric and antisymmetric waves were determined.

1. Consider a piezoelectric strip of symmetry class 6 mm, occupying the region $|z|<2h$, $|x|<\infty$, $|y|<\infty$, assuming that electrodes having potentials which vary sinusoidally with time are situated on the surfaces $z=\pm h$, free from mechanical loads, and the axis of symmetry of the medium coincides with the z axis of the chosen system of coordinates (Fig. 1).

The equations of electro-elasticity for determining the amplitudes of the displacements u and w and the potential of the electric field φ in the case of plane strain can be represented in the following form (the time factor $\exp(-i\omega t)$ is omitted)

$$\begin{gathered}
\frac{\partial^2 u}{\partial x^2}+\mu_1\frac{\partial^2 u}{\partial z^2}+l^2u+\mu_2\frac{\partial^2 w}{\partial x\partial z}+\frac{\partial^2\varphi}{\partial x\partial z}=0\\
\mu_2\frac{\partial^2 u}{\partial x\partial z}+\mu_1\frac{\partial^2 w}{\partial x^2}+\mu_4\frac{\partial^2 w}{\partial z^2}+l^2w+\mu\ \frac{\partial^2\varphi}{\partial z^2}+\mu_6\frac{\partial^2\varphi}{\partial z^2}=0\\
-\left(k_1^2\frac{\partial^2 u}{\partial x\partial z}+k_2\frac{\partial^2 w}{\partial x^2}+k_3\frac{\partial^2 w}{\partial z^2}\right)+\mu_7^2\frac{\partial^2\varphi}{\partial x^2}+\frac{\partial^2\varphi}{\partial z^2}=0
\end{gathered}\tag{1.1}$$

For symmetric Lamb waves we can write the solution of system (1.1) in the form

$$\left\|\begin{matrix}u(x,z)\\ w(x,z),\varphi(x,z)\end{matrix}\right\|=2\int\left\|\begin{matrix}U(p)\\ W(p),\Phi(p)\end{matrix}\right\|\left\|\begin{matrix}\operatorname{ch}(\lambda ph)\\ \operatorname{sh}(\lambda ph)\end{matrix}\right\|\left\|\begin{matrix}\cos(px)\\ \sin(px)\end{matrix}\right\|dp\tag{1.2}$$

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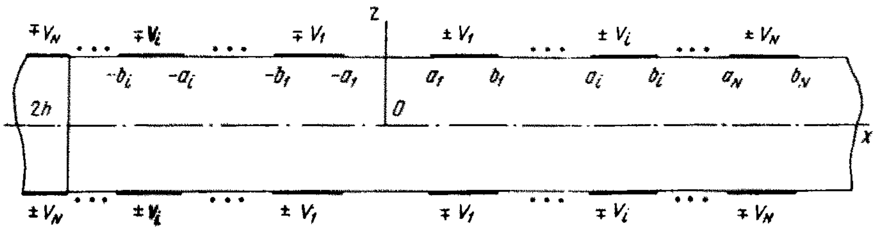

FIG. 1.

$$\mu_1 = \frac{c_{44}^E}{c_{11}^E}, \quad \mu_2 = \frac{c_{13}^E + c_{44}^E}{c_{11}^E}, \quad \mu_4 = \frac{c_{33}^E}{c_{11}^E}, \quad \mu_5 = \frac{e_{33}}{e_{31} + e_{15}}, \quad \mu_6 = \frac{e_{15}}{e_{31} + e_{15}}$$

$$\mu_7^2 = \frac{\varepsilon_{11}^S}{\varepsilon_{33}^S}, \quad k_1^2 = \frac{(e_{31} + e_{15})^2}{c_{11}^E \varepsilon_{33}^S}, \quad k_2 = \mu_6 k_1^2, \quad k_3 = \mu_5 k_1^2, \quad l = \frac{\omega L}{V_x}, \quad V_x^2 = \frac{c_{11}^E}{\rho}$$

$$x_* = \frac{x}{L}, \quad z_* = \frac{z}{L}, \quad u_* = \frac{u}{L}, \quad w_* = \frac{w}{L}, \quad \varphi_* = \frac{\varphi}{L}, \quad \gamma_0 = \frac{c_{11}^E L}{e_{31} + e_{15}}$$

We will henceforth omit the asterisks, L is the characteristic linear dimension of the problem (for example, the width of the electrode or the thickness of the strip), and c_{ij}^E, ε_{ij}^S, e_{ij}, ρ are the characteristics of the material of the strip.

When antisymmetric oscillations are excited, we must interchange ch $(\lambda p z)$ and sh $(\lambda p z)$ in (1.2).

Substituting (1.2) into (1.1), we obtain a homogeneous system of linear algebraic equations for the unknown functions U, W and φ a non-zero solution of which exists provided that λ satisfies the characteristic equation

$$\det\| a_{ij} \| = 0 \tag{1.3}$$

$$a_{11} = 1 - \mu_1\lambda^2 - \nu^2, \quad a_{12} = a_{21} = -\mu_2\lambda, \quad a_{13} = -\lambda, \quad a_{22} = -\mu_1 + \mu_4\lambda^2 + \nu^2$$

$$a_{23} = \mu_5\lambda^2 - \mu_6, \quad a_{31} = k_1^2\lambda, \quad a_{32} = k_2 - k_3\lambda^2, \quad a_{33} = \lambda^2 - \mu_7^2, \quad \nu = l/p$$

Equation (1.3) is a bicubic equation, and its properties have been investigated in [5]. Taking these results into account as well as the symmetry of the electro-elastic fields with respect to the variable z, we have for the displacements and potential

$$u(x,z) = 2\int \sum_j \alpha_j \operatorname{ch}(\lambda_j p z) U_j(p) \cos(px)\, dp$$

$$w(x,z) = 2\int \sum_j \beta_j \operatorname{sh}(\lambda_j p z) U_j(p) \sin(px)\, dp \tag{1.4}$$

$$\varphi(x,z) = 2\int \sum_j \gamma_j \operatorname{sh}(\lambda_j p z) U_j(p) \sin(px)\, dp$$

$$\alpha_j = \alpha_j(\lambda_j) = a_{12}a_{23} - a_{13}a_{22}, \quad \beta_j = \beta_j(\lambda_j) = a_{21}a_{13} - a_{11}a_{23},$$

$$\gamma_j = \gamma_j(\lambda_j) = a_{11}a_{22} - a_{12}^2$$

Here and everywhere henceforth the integration with respect to p is carried out from 0 to ∞, and the summation over j is carried out from $j = 1$ to $j = 3$.

The potential of the electric field φ_v in a region of vacuum $z < -h$ with permittivity ε is given by the expression

$$\overline{\varphi} = 2\int \overline{\Phi}(p)\exp(pz)\sin(px)dp, \quad \overline{\varphi} = \frac{\varphi_\upsilon}{\gamma_0} \tag{1.5}$$

Using the equations of state [6] we will determine the components of the mechanical stresses from the equations

$$\begin{aligned}
\sigma_{xx} &= 2c_{11}^E \int p \sum_j t_j^* U_j(p)\operatorname{ch}(\lambda_j pz)\sin(px)dp \\
\sigma_{zz} &= 2c_{33}^E \int p \sum_j m_j^* U_j(p)\operatorname{ch}(\lambda_j pz)\sin(px)dp \\
\sigma_{xz} &= 2c_{44}^E \int p \sum_j n_j^* U_j(p)\operatorname{sh}(\lambda_j pz)\cos(px)dp
\end{aligned} \tag{1.6}$$

$$t_j^* = -\alpha_j + (\mu_2 - \mu_1)\beta_j\lambda_j + (1-\mu_6)\gamma_j\lambda_j, \quad n_j^* = \alpha_j\lambda_j + \beta_j + (\mu_6/\mu_1)\gamma_j$$

$$m_j^* = -\mu_8\alpha_j + (\mu_5/\mu_4)\gamma_j\lambda_j + \beta_j\lambda_j, \quad \mu_8 = c_{13}^E / c_{33}^E$$

The boundary conditions $\sigma_{zz}(x, -h) = \sigma_{xz}(x, -h) = 0$ will be satisfied when

$$U_j(p) = \Delta_j(p)U_0(p) \quad (j = 1, 2, 3) \tag{1.7}$$

$$\Delta_1 = m_2n_3 - m_3n_2, \qquad \Delta_2 = m_3n_1 - m_1n_3, \qquad \Delta_3 = m_2n_2 - m_2n_1$$

$$m_j = m_j^* \operatorname{ch}(\lambda_j ph), \quad n_j = n_j^* \operatorname{sh}(\lambda_j ph)$$

Taking the relations (1.4), (1.5) and (1.7) and the condition of continuity of the electric-field potential when $z = -h$ into account, we have

$$\overline{\Phi}(p) = -\sum_j \gamma_j^* \Delta_j U_0(p)\exp(ph), \quad \gamma_j^* = \gamma_j \operatorname{sh}(\lambda_j ph) \tag{1.8}$$

Before considering the solution of the initial problem we will consider the auxiliary problem in which a specified jump in charge density $\sigma(x)$ is distributed on the surface $z = -h$

$$D_z - \overline{D}_z = \sigma(x) \tag{1.9}$$

By defining the components D_z, $\overline{D}_z$ of the electric-induction vectors in the piezoelectric medium and in a vacuum, by the relations

$$\begin{aligned}
D_z &= 2\int p \sum_j q_j\Delta_j \operatorname{ch}(\lambda_j pz)U_0(p)\sin(px)dp \\
\overline{D}_z &= 2\int p \sum_j \gamma_j^*\Delta_j U_0(p)\exp(z+h)\sin(px)dp
\end{aligned} \tag{1.10}$$

we obtain from (1.9) for $U_0(p)$

$$\pi p R_{21}(p)U_0(p) = \mu_9 \int \sigma(x)\sin(px)dp \tag{1.11}$$

$$R_{21}(p) = R_2(p) - \varepsilon_0 R_1(p), \quad \{R_1(p), R_2(p)\} = \sum_j \{\gamma_j^*, q_j^*\}\Delta_j, \quad \varepsilon_0 = \frac{\varepsilon}{\varepsilon_{33}^S}$$

$$q_j = -\gamma_j\lambda_j - e_{31}\mu_9\alpha_j + e_{33}\mu_9\beta_j\lambda_j, \quad q_j^* = q_j \operatorname{ch}(\lambda_j ph), \quad \mu_9 = \frac{e_{31} + e_{15}}{c_{11}^E \varepsilon_{33}^S}$$

Taking (1.4), (1.7) and (1.11) into account we obtain for the electric-field potential on the boundary $z = -h$

$$\pi\varphi(x, -h) = -\mu_9 \int \sigma(t)K(t, x)dt \tag{1.12}$$

$$K(t,x)=2\int\frac{R_1(p)}{pR_{21}(p)}\sin(pt)\sin(px)dp$$

Since in the problem considered (Fig. 1) $\sigma(x)=0$ on the non-electrode parts, while the value of the potential is known on the electrodes, from (1.12) we have the following system of integral equations for determining the jump in the charge density at each electrode

$$\sum_{k=1}^{N}\int_{a_k}^{b_k}\sigma_k(t)K(t,x)dt=\pi e_i\quad(a_i<x<b_i,\ i=1,2,\ldots,N),\quad e_i=-V_i\varepsilon_{33}^S/L \tag{1.13}$$

2. The kernel $K(t, x)$ contains a logarithmic discontinuity, which can be isolated by taking into account the asymptotic properties of the roots of Eq. (1.3), established in [5]. Without going into the details of the calculations, we note that as $p\to\infty$

$$\begin{Bmatrix}R_1(p)\\R_2(p)\end{Bmatrix}\sim\begin{Bmatrix}-i\gamma\\iq\end{Bmatrix}\exp[-(\lambda_1+2\lambda)ph](m_{21}^*n_{22}^*-m_{22}^*n_{21}^*)/4 \tag{2.1}$$

where λ_1 is a negative real root, and λ is the real part (negative) of the complex-conjugate root of Eq. (1.3).

Relations (2.1) enable one to convert (1.13) to the following form

$$\sum_{k=1}^{N}\int_{a_k}^{b_k}\sigma_k(t)\ln\left|\frac{x+t}{x-t}\right|dt+\sum_{k=1}^{N}\int_{a_k}^{b_k}\sigma_k(t)K_*(t,x)dt=\pi\upsilon_i^* \tag{2.2}$$

$$(a_i<x<b_i,\ i=1,2,\ldots,N),\quad \upsilon_i^*=-e_i(q_1^\infty+\varepsilon_0\gamma_1^\infty)/\gamma_1^\infty,$$

$$\{q_1^\infty,\gamma_1^\infty\}=\lim_{p\to\infty}\{\gamma_1(p),\ q_1(p)\}$$

$$K_*(t,x)=-2\int\frac{q_1^\infty R_1(p)+\gamma_1^\infty R_2(p)}{\gamma_1^\infty R_{21}(p)p}\sin(pt)\sin(px)dp$$

Representing the charge density on an arbitrary electrode in the form of series in Chebyshev polynomials of the first kind

$$\sigma_k(\tau)=\frac{\upsilon_i^*}{\sqrt{1-\tau^2}}\sum_{m=0}^{\infty}A_m^{(k)}\frac{T_m(\tau)}{\xi_{1k}} \tag{2.3}$$

and carrying out the Bubnov–Galerkin procedure, we obtain the following N infinite systems of equations from (2.2) for determining the coefficients $A_m^{(k)}$

$$\sum_{k=1}^{N}\sum_{m=0}^{\infty}A_m^{(k)}(\alpha_{ms}^{(ki)}+\gamma_{ms}^{(ki)})=\upsilon_i^{(0)}\delta_s,\quad \upsilon_i^{(0)}=\frac{\upsilon_i^*}{\upsilon_1^*} \tag{2.4}$$

$$(s=0,1,2,\ldots,i=1,2,\ldots,N)$$

The coefficients of (2.4) can be found from the formulae

$$\alpha_{ms}^{(ki)}=\frac{2}{\pi^2}\int_{-1}^{1}\int_{-1}^{1}\frac{T_m(\tau)T_s(\eta)}{\sqrt{1-\tau^2}\sqrt{1-\eta^2}}\ln\left|\frac{\xi_{1i}\eta+\xi_{1k}\tau+\xi_{2i}+\xi_{2k}}{\xi_{1i}\eta-\xi_{1k}\tau+\xi_{2i}-\xi_{2k}}\right|d\tau d\eta$$

$$\gamma_{ms}^{(ki)}=-\int\frac{q_1^\infty R_1(p)+\gamma_1^\infty R_2(p)}{\gamma_1^\infty R_{21}(p)p}S_m(\xi_{2k}p)S_s(\xi_{2i}p)J_m(\xi_{1k}p)J_s(\xi_{1i}p)dp \tag{2.5}$$

$\delta_s = 2\delta_{s0}$, $\delta_{00} = 1$, $\delta_{s0} = 0$ $(s > 0)$, $J_m(\cdot)$ are Bessel functions and

$$S_m(\cdot) = [(1-(-1)^m)(-1)^{(m-1)/2}\cos(\cdot) + (1+(-1)^m)(-1)^{m/2}\sin(\cdot)]$$

The variables τ and η in (2.4) are connected with t and x by the following relations

$$t = \xi_{1k}\tau + \xi_{2k}, \quad x = \xi_{1i}\eta + \xi_{2i}, \quad 3\xi_{1k} = b_k + a_k, \quad 2\xi_{2k} = b_k - a_k \qquad (2.6)$$

The roots of the following equation are of importance later

$$R_{21}(p) = 0 \qquad (2.7)$$

the zeros of which are the poles of the kernel $K(t, x)$.

In Fig. 2 we show the behaviour of the first five roots of Eq. (2.7), which are not the same as the roots of the equation $R_1(p) = 0$, as a function of the parameter $h_* = h/\lambda_x$ $(\lambda_x = 2\pi V_x/\omega$, $V_x^2 = c_{11}^E/\rho)$ for a strip of cadmium sulphide [7]. Numerical calculations showed that when h_* increases the least root of Eq. (2.7) approaches the value $\nu = \nu_R = 0.3993$, which defines the velocity of the Rayleigh wave, and subsequent roots approach the value $\nu = \nu_1 = 0.414$, corresponding to the velocity of a volume shear wave.

Since Eq. (2.7) has a finite number of zeros for any h_*, when evaluating the integrals in (2.5) one must take into account the contribution of the poles, while the integrals are understood in the sense of the principal value.

After solving system (2.4) all the characteristics of the electro-elastic field in the strip can be determined. In particular, using relations (1.4), (1.7), (1.11) and (2.3) for the displacements and the potential we obtain

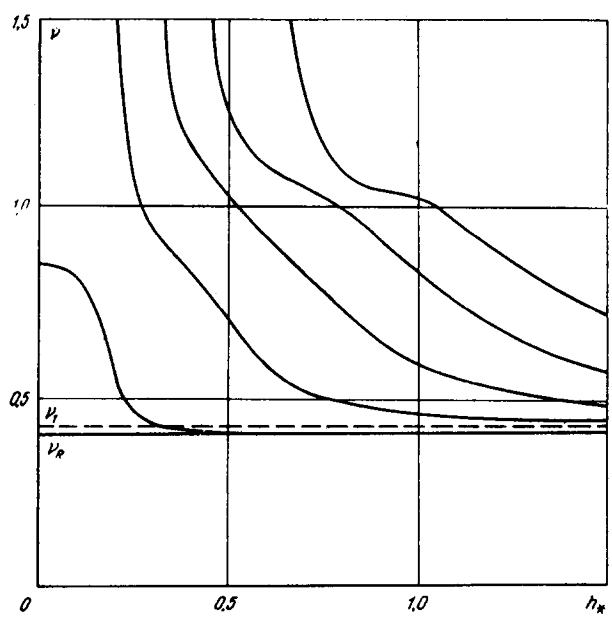

FIG. 2.

$$\begin{Bmatrix} u(x,z) \\ w(x,z) \\ \varphi(x,z) \end{Bmatrix} = \Gamma_0 \begin{Bmatrix} E_0 \\ E_0 \\ V_1 \end{Bmatrix} \int \sum_j \Delta_j \begin{Bmatrix} \alpha_j \operatorname{ch}(\lambda_j pz) \\ \beta_j \operatorname{sh}(\lambda_j pz) \\ \gamma_j \operatorname{sh}(\lambda_j pz) \end{Bmatrix} \frac{Q(p)\sin(px)}{pR_{21}(p)} dp \tag{2.8}$$

$$Q(p) = \sum_{k=1}^{N} Q^{(k)}(p), \quad Q^{(k)}(p) = \sum_{m=0}^{\infty} A_m^{(k)} S_m(\xi_{2k}p) J_m(\xi_{1k}p)$$

$$E_0 = (e_{31} + e_{15})(c_{11}^E L)^{-1} V_1, \quad \Gamma_0 = (q_1^\infty + \varepsilon_0 \gamma_1^\infty)/\gamma_1^\infty$$

When evaluating the integrals in (2.8) one must take into account all the poles of the integrands. Taking the time factor $\exp(-i\omega t)$ into account, we obtain from (2.8), by calculations similar to those in [5–7]

$$2\begin{Bmatrix} u(x,z,t) \\ w(x,z,t) \\ \varphi(x,z,t) \end{Bmatrix} = \Gamma_0 \begin{Bmatrix} E_0 \\ -iE_0 \\ -iV_1 \end{Bmatrix} V \cdot p \cdot \int \sum_j \Delta_j \begin{Bmatrix} \alpha_j \operatorname{ch}(\lambda_j pz) \\ \beta_j \operatorname{sh}(\lambda_j pz) \\ \gamma_j \operatorname{sh}(\lambda_j pz) \end{Bmatrix} \frac{Q(p)H(px,t)}{pR_{21}(p)} dp + \begin{Bmatrix} u_L \\ w_L \\ \varphi_L \end{Bmatrix} \tag{2.9}$$

$$H(\cdot) = H^+(\cdot) + H^-(\cdot), \quad H^\pm = \exp(\mp i(px \mp \omega t))$$

The functions u_L, w_L and φ_L in (2.9) represent electro-elastic Lamb waves propagating in the positive and negative directions of the x axis, and can be calculated from the formulae

$$2\begin{Bmatrix} u_L \\ w_L \\ \varphi_L \end{Bmatrix} = \pi\Gamma_0 \begin{Bmatrix} iE_0 \\ E_0 \\ V_1 \end{Bmatrix} \sum_{r=1}^{P} \sum_j \Delta_{jr} \begin{Bmatrix} \alpha_{jr} \operatorname{ch}(\lambda_{jr} p_r z) \\ \beta_{jr} \operatorname{sh}(\lambda_{jr} p_r z) \\ \gamma_{jr} \operatorname{sh}(\lambda_{jr} p_r z) \end{Bmatrix} \frac{Q(p_r)H(p_r x,t)}{p_r R_{21}'(p_r)} \tag{2.10}$$

We can similarly determine the other components of the conjugate electro-elastic field. For example, for the stresses σ_{xx} and σ_{xz}, related to the Lamb waves, we will have

$$2\begin{Bmatrix} \sigma_{xxL} \\ \sigma_{xzL} \end{Bmatrix} = \pi\Gamma_0 \begin{Bmatrix} E_0 \\ -iE_0 \end{Bmatrix} \sum_{r=1}^{P} \sum_j \Delta_{jr} \begin{Bmatrix} t_{jr}^* \operatorname{ch}(\lambda_{jr} p_r z) \\ n_{jr}^* \operatorname{sh}(\lambda_{jr} p_r z) \end{Bmatrix} \frac{Q_r(p_r)H(p_r x,t)}{R_{21}'(p_r)} \tag{2.11}$$

$$R_{21}' = \left.\frac{dR_{21}(p)}{dp}\right|_{p=p_r}$$

In relations (2.10) and (2.11) we have denoted the number of poles by P, and the subscript r on the quantities Δ_{jr}, α_{jr}, ..., n_{jr}^* indicates that these quantities are calculated for $p = p_r$ (p_r is the root of Eq. (2.7)).

3. The solution of the problem of the antisymmetric Lamb waves is constructed in the same way as in the case considered above, but here the functions $R_1(p)$ and $R_2(p)$ are defined as follows:

$$\begin{Bmatrix} R_1(p) \\ R_2(p) \end{Bmatrix} = \sum_j \begin{Bmatrix} \gamma_j^* \\ q_j^* \end{Bmatrix} \Delta_j, \quad \gamma_j^* = \gamma_j \operatorname{ch}(\lambda_j ph), \quad q_j^* = q_j \operatorname{sh}(\lambda_j ph) \tag{3.1}$$

where λ_j are the roots of the characteristic equation (1.3).

The propagation velocities of the antisymmetric Lamb waves in the strip are given by the roots of Eq. (2.7) in which the functions $R_1(p)$ and $R_2(p)$ are defined by (3.1). The dependence of the first four roots of this equation on h is shown in Fig. 3 for a strip of CdS [3].

The electro-elastic fields related to the symmetric Lamb waves propagating in the strip can be obtained from (2.10) and (2.11) by replacing $\mathrm{ch}(\lambda_{jr}p_r z)$ by $-\mathrm{sh}(\lambda_{jr}p_r z)$ and $\mathrm{sh}(\lambda_{jr}p_r z)$ by $-\mathrm{ch}(\lambda_{jr}p_r z)$.

4. The results obtained were used for a numerical investigation of the distribution of the characteristics of the electro-elastic field along the $z_* = z/h$ coordinate. The calculations were carried out for a strip of CdS. In Figs 4–8 we show the calculated values of the real parts (the continuous curves) and the imaginary parts (the dashed curves) of the functions

$$\begin{Bmatrix} U_L \\ W_L \\ \Phi_L \end{Bmatrix} = \sum_{r=1}^{P} \sum_{j} \Delta_{jr} \begin{Bmatrix} \alpha_{jr}\,\mathrm{ch}(\lambda_{jr}p_r z) \\ \beta_{jr}\,\mathrm{sh}(\lambda_{jr}p_r z) \\ \gamma_{jr}\,\mathrm{sh}(\lambda_{jr}p_r z) \end{Bmatrix} \frac{Q(p_r)}{p_r R'_{21}(p_r)}$$

$$\begin{Bmatrix} \Sigma_{xx} \\ \Sigma_{xz} \end{Bmatrix} = \sum_{r=1}^{P} \sum_{j} \Delta_{jr} \begin{Bmatrix} t^*_{jr}\,\mathrm{ch}(\lambda_{jr}p_r z) \\ n^*_{jr}\,\mathrm{sh}(\lambda_{jr}p_r z) \end{Bmatrix} \frac{Q(p_r)}{R'_{21}(p_r)}$$

over the thickness of the strip $z_* = z/h$, where the curves denoted by the number "1" refer to the case when $h_* = h/\lambda_x = 0.05$, and the curves denoted by the number "2" refer to the case when $h = 0.1$ and $\lambda_x = 10^{-3}$ m.

When carrying out the calculations in (2.5) we confined ourselves to a 3×3 system and assumed that $N = 1$ (two oppositely charged electrodes on each surface of the strip).

The numerical data confirm that for small h_* the distribution of the mechanical fields in the far zone agrees with the Kirchhoff–Love hypotheses.

The potential distribution and the components D_x over the thickness of the plate is close to linear, while the components D_z are close to uniform.

The distribution of the electro-elastic fields over the thickness of the strip becomes extremely non-uniform as the parameter h_* increases.

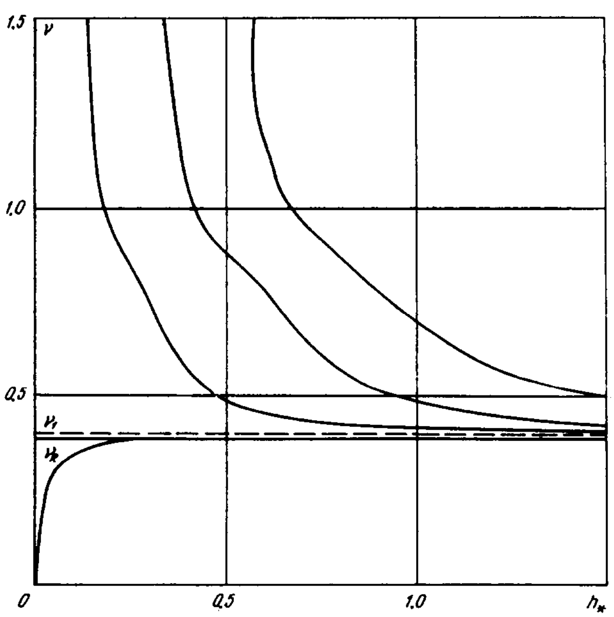

FIG. 3.

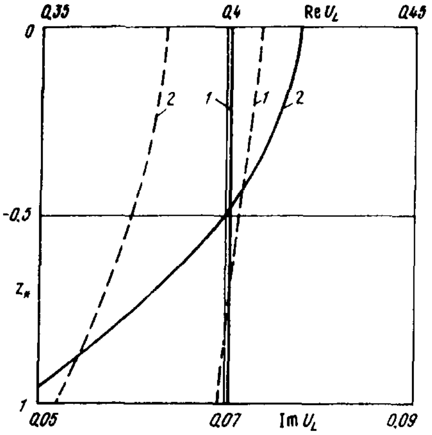

FIG. 4.

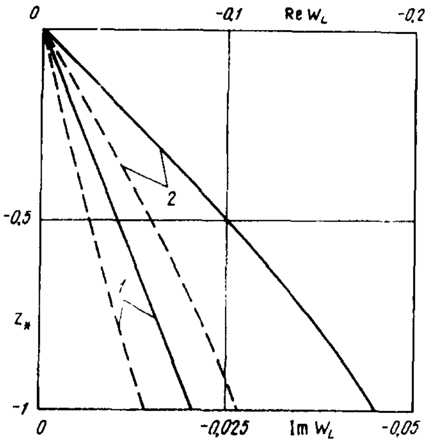

FIG. 5.

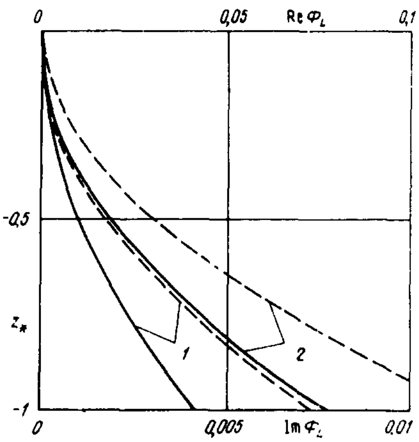

FIG. 6.

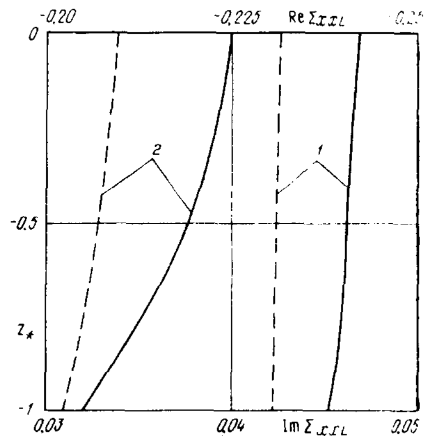

FIG. 7.

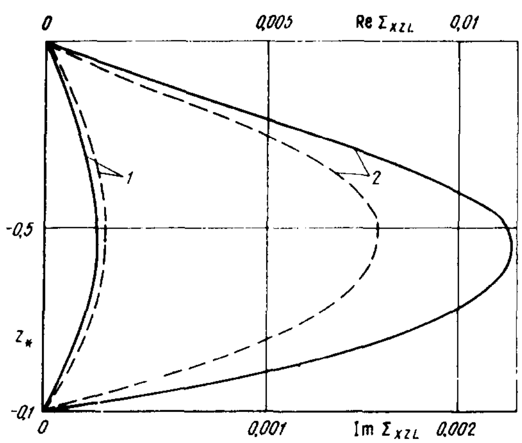

FIG. 8.

Similar calculations were carried out for a symmetrical connection of electrodes, when antisymmetric Lamb waves are excited in the strip. We established by numerical calculations for $h_* < 0.05$ that the distribution of the displacement W_L is close to uniform, while U_L and Σ_{xx} are close to linear. It should be noted that here the distribution of the function Σ_{xx}, proportional to the shear stress σ_{xz}, over the thickness of the strip is described by a second-degree polynomial in the variable z, and the maximum of the shear stress is reached on the axis of the strip and considerably exceeds the maximum value of the stress σ_{xx} reached at the boundaries $z = \pm h$ of the strip.

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